NUMERICAL EVALUATION OF SHEAR STRESS DISTRIBUTION ON SHIP CROSS SECTION Part 2

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SUMMARY

Shear stress requirement of BKI *Rules for Hull* (Vol.II,Sect.5) for hull girder assessment when incomplete hull analysis is performed consist of shear stress caused by stillwater and wave bending both of sagging and hogging, horizontal bending, static and wave torsion both of st.venant and warping. This study extending basic idea for numerical procedure of normal stress distribution in Part-1 study, and that not about understanding the theory behind stresses caused by wave bending nor about deriving complicated equation for warping stress, but it will introduce more practical solution with a detailed step-by-step for evaluating the shear stress distribution on ship cross section. Finally the proposed procedure is a reliable and useful tool for computing the shear stresses distribution, which does not need a sophisticated computer software

1. INTRODUCTION

Longitudinal strength of ship is the most of important aspect in ship structural design, since whole ship's structures subjected by various loading like bending, shearing and torsion, she became the most concern of naval architect including Indonesian national classification society (BKI).

BKI give minimum standard of plate thicknesses as the function of hull girder strength, therefore global design stress for determine hull girder became one thing needed most of all. Several studies have been carried out to explain the formula and derived complex global design stress equation, but just few that discuss about the practical solution. It can be found in (a)Vernon and Nadeau (1987), which is described unified developments of the St-Venant and warping-based on thin-walled beam theories and their application in the torsional analysis of ship structures, (b) Hu (1992) described the mathematical derivation of the equation used for the calculation of cross sectional constants and stress distributions of thin-walled sections, and (c) Lue et al.(2007) have been proposed a numerical procedure for computing the warping constant for an arbitrary cold-formed steel open section.

In Part 1-study, numerical procedure was performed to investigate normal stress distribution on arbitrary ship cross section. A set of step by step practical solution was carried out, and proven became reliable and useful tool for computing normal stress distribution, which does not need a sophisticated computer software.

In this study, the numerical procedure extended for calculate shear stress distribution on any arbitrary type of cross section, most of calculation procedure in Part-2 derived from Part-1, so the procedure that has been discussed in Part-1 won't be repeated, except a few to make clear connection between both of those two part. The requirement of hull girder assessment depend on BKI Rules for Hull, Vol II, Section 5.

2. HULL GIRDER STRESS RESPONSE

Hull girder strength described ship as a beam composed of plates, stiffener and girders, large ratio between length of ship and its plate thicknesses need thin wall theory in order to analyse the stress responses. The classical theory of thin-walled beams with arbitrary open cross section have been described briefly in Part-1 study.

2.1. Bending without Twist

Called from Part-1 study, equation (14) become equation (1) in this Part-2 study:

$$I_{yy} = \int_0^b x \cdot x \cdot t \cdot ds \text{ , with statical moment, } S_y = \int x \cdot t \cdot ds$$

$$I_{yy} = -\int_0^b S_y \cdot dx \qquad (1)$$

in the same way Ixx, Ixy, and Iyx respectively can be expressed as:

$$I_{xx} = -\int_0^b S_x \cdot dy \quad I_{xy} = -\int_0^b S_x \cdot dy \quad I_{yx} = -\int_0^b S_y \cdot dx \qquad (2)$$

The shear stress from equilibrium of the slice of the cut section from figure 1 can be written as:

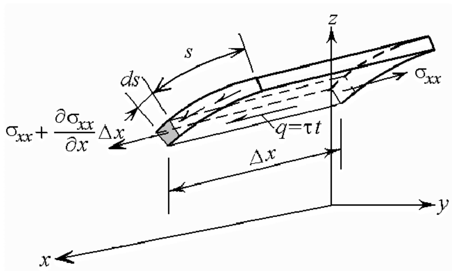


Figure 1: Equilibrium of cut section

$$\tau_y = \frac{-\left(S_y \cdot I_{xx} - S_x \cdot I_{xy}\right)}{\left(-I_{xy}^2 + I_x \cdot I_y\right)} \cdot \frac{S_{FH}}{t} \text{ and}$$

$$\tau_x = \frac{-\left(S_x \cdot I_{yy} - S_y \cdot I_{xy}\right)}{\left(-I_{xy}^2 + I_x \cdot I_y\right)} \cdot \frac{S_{FV}}{t}$$

For symmetry section $I_{xy} = 0$, the equation become:

$$\tau_y = \frac{-\left(S_y \cdot I_{xx}\right)}{\left(I_x \cdot I_y\right)} \cdot \frac{S_{FH}}{t} \text{ and}$$

$$\tau_x = \frac{-\left(S_x \cdot I_{yy}\right)}{\left(I_x \cdot I_y\right)} \cdot \frac{S_{FV}}{t} \qquad (4)$$

Where S_{FH} is shear force in horizontal direction and S_{FV} is shear force in vertical direction.

2.2. Warping Stress

Called from Part-1 study, equation (20) become equation (5) in this Part-2 study:

$$\omega_n = \frac{\int \left(\omega_o - \omega_2\right) \cdot dA}{\int dA} \qquad (5)$$

Then warping static torsion can be written as:

$$S\grave{\omega} = \int \grave{\omega} n.t \; dS \qquad (6)$$

Call from Part-1 study, compatibility strain equation (2):

$$\frac{d}{ds} u + \frac{d}{dx} v = \gamma = 0$$

The warping shear flow can be obtained from above equation as:

$$q\grave{\omega} = E \cdot \varphi''' \int \grave{\omega} \cdot t \cdot dS \qquad (7)$$

Then shear stress for warping mode can be written as follow:

$$\tau_\omega = \frac{S_\omega \cdot T_\omega}{I_\omega \cdot t} \qquad (8)$$

Where $T\grave{\omega}$ is torsional load and $I\grave{\omega}$ is warping constant that has been described in equation (23) in Part-1 study.

3. NUMERICAL FORMS FOR SHEAR STRESS DISTRIBUTION

Hu (1992) and Lue,et.al. (2007) had been studying numerical procedure of shear stresses distribution on longitudinal section, and it will be summarized in this study.

3.1. Bending Shear Stress

Equation (3) showed that pure bending without twist on cross section could be divided into the vertical and horizontal shear stresses distribution. Where the shear flow distribution on each segment comprised of 3 nodal coordinates (i, k, j) and were described below:

For horizontal shear flow:

$$q_{y,ikj} = \frac{-\left(S_{y,ikj} \cdot I_{xx} - S_{x,ikj} \cdot I_{xy}\right)}{\left(-I_{xy}^2 + I_x \cdot I_y\right)} \text{ and}$$

For vertical shear flow:

$$q_{x,ikj} = \frac{-\left(S_{x,ikj} \cdot I_{yy} - S_{y,ikj} \cdot I_{xy}\right)}{\left(-I_{xy}^2 + I_x \cdot I_y\right)} \qquad (9)$$

And the bending shear stress will be calculate as follow:

$$\tau_{x,ikj} = q_{x,ikj} \cdot \frac{S_{FH}}{t} \text{ for horizontal direction and}$$

$$\tau_{x,ikj} = q_{x,ikj} \cdot \frac{S_{FV}}{t} \text{ for vertical direction} \qquad (10)$$

3.2. Warping Shear Stress

Warping shear stress was started from define warping static torsion. Warping static torsion has the same pattern as static torsion (Sx, Sy) while calculated, the value will be zero (0) in free end including the free end from cut procedure in close form, and also has flow and sum of flow in accordance to the calculation that have been carried out on static torsion.

Warping static torsion for each nodal coordinates will follow:

$$S\grave{\omega},i = S\grave{\omega},j$$
$$S\grave{\omega},k = S\grave{\omega},i + Ak \cdot (\grave{\omega}n,i + \grave{\omega}n,k) / 4$$
$$S\grave{\omega},j = S\grave{\omega},i + Ak \cdot (\grave{\omega}n,i + \grave{\omega}n,j) / 2 \qquad (11)$$

Then warping shear flow for each nodal coordinates will be conducted as follow:

$$q_{\omega,ikj} = \frac{S_{\omega,ikj}}{I_\omega} \qquad (12)$$

And for the end, the warping shear stress could be counted by:

$$\tau_{\omega,ikj} = q_{\omega,ikj} \cdot \frac{T_{\omega}}{t} \qquad (13)$$

3.3. Shear Flow Correction for Close Form

All of shear stress distribution from above procedure would be corrected (in shear flow form) if it has close section form, either single or multi cell. Numerical form of correction procedure valid for all of shear stress distribution, where those numerical procedure were derived from this equation:

$$q = q - q_{corr} = q - \left[\frac{\oint \left(\frac{q}{t} \right) \cdot dS}{\oint \left(\frac{1}{t} \right) \cdot dS} \right] \qquad (14)$$

Where $\{1: \oint(1/t)dS\}$ is matrices $[K]^{-1}$, and that has been described in equation (32) Part-1 study, while $\{\int(q/t)dS\}$ consist of 3 nodal coordinates, as follow :

$\int \left(\frac{q}{t} \right) \cdot dS = \frac{1}{6} \cdot \left(q_i + 4 \cdot q_k + q_j \right) \cdot \frac{b}{t}$ and

$$\oint \left(\frac{q}{t} \right) \cdot dS = \sum \int \left(\frac{q}{t} \right) \cdot dS \text{ for each cell} \qquad (15)$$

4. NUMERICAL STUDY

Ship cross section with a combination of open, single closed and multicell would be used to describe the proposed numerical procedure. Its principal dimension was showed in Table 1 included with detailed model was represented in Figure 2.

Tabel 1 : Principal Partikular

Length between perpendicular	Lpp	:	110	m
Length between waterline	Lwl	:	110	m
Length of Scantling	L	:	106.7	m
Depth	H	:	15	m
Breadth	B	:	20	m
Draft	T	:	10	m
Speed	v	:	6	kn
Block Coefficient	Cb	:	0.9	

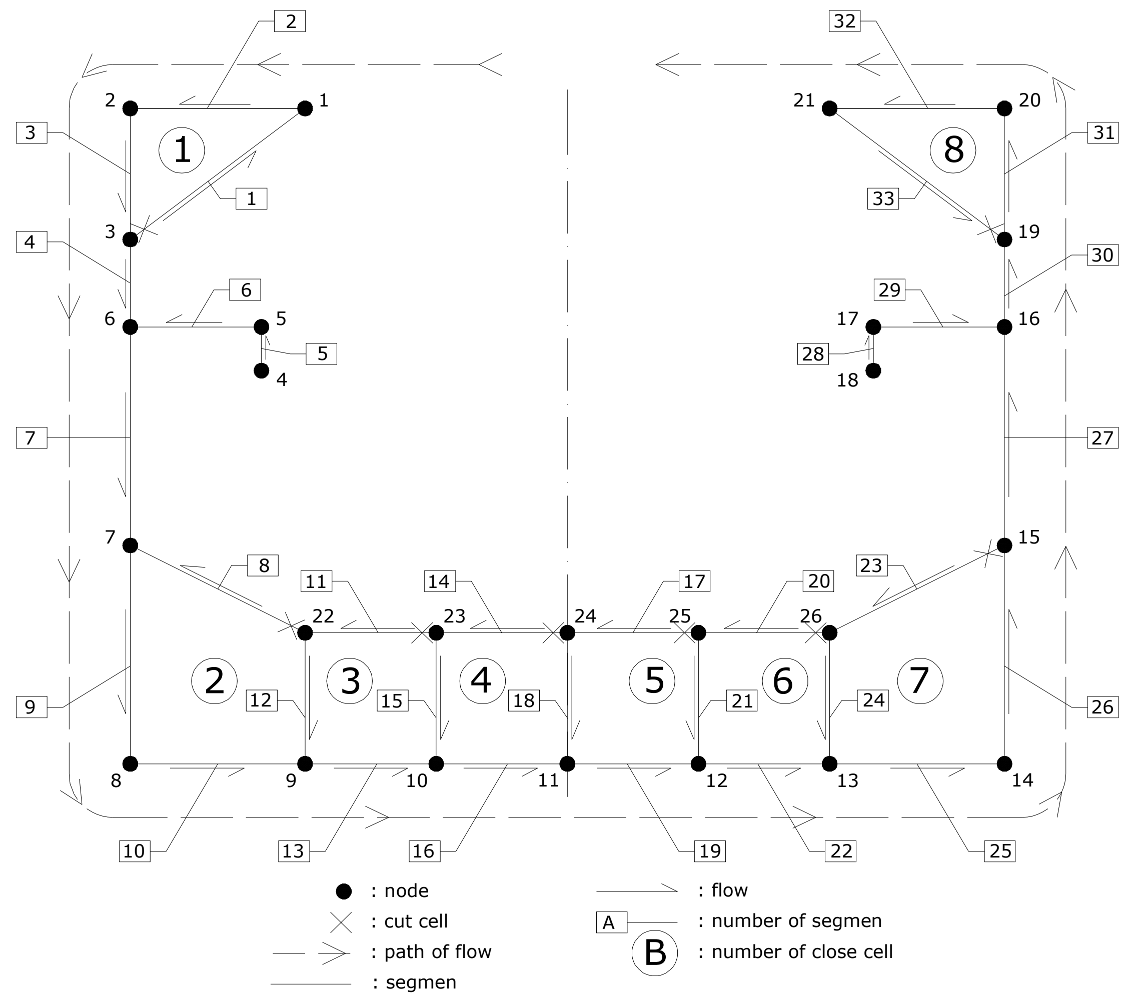


Figure 2 : Sample Model with combination open, single close, multi cell

The numerical procedure of sample model will be described globally in sixth steps, and that is:

Step (1)
Calculation of hull girder load for the model was done and determined according to BKI Rules for Hull. Figure 3 illustrated the distribution of Vertical Shear force hogging condition (S_{FV+}) and sagging condition (S_{FV-}), Horizontal Shear force (S_{FH}), and Wave torsional bending (M_{WT}).

Step (2)
Call Part-1 study, Table 2 give an information about static moment (Sx or Sy) and the characteristics of moment inertia of cross section. Where the values of moment of inertia x with respect to neutral axis (INA-x = 48.03), moment of inertia y with respect to neutral axis (INA-y = 93.36) and product moment of inertia x and y ($I_{xy} = I_{yx} = 0.00$). Zero values of $I_{xy} = I_{yx}$ because of symmetrical form of cross section was evaluated.

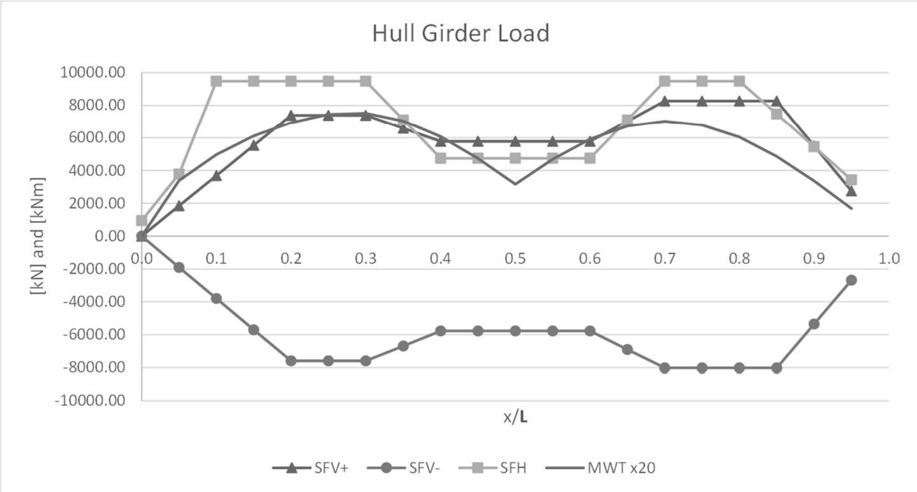


Figure 3: Shear Force and Torsional Load

Table 2: Static moment (S_X and S_Y) and Moment of Inertia (I_{NA-X}, I_{NA-Y}, and I_{NA-XY})

Node ID	X [m]	Y [m]	Seg.	Node i	Node j	thick. [m]	Sxi' [m³]	Sxk' [m³]	Sxj' [m³]	Product-x	I_{NA}-x [m⁴]	I_{NA}-xy [m⁴]	Syi' [m³]	Syk' [m³]	Syj' [m³]	Product-y	I_{NA}-y [m⁴]	I_{NA}-yx [m⁴]
1	-6	15	1	3	1	0.015	0.00	0.26	0.58	1.62	-0.81	-1.08	0.00	-0.34	-0.60	-1.95	1.30	0.98
2	-10	15	2	1	2	0.015	0.58	0.85	1.13	5.13	0.00	3.42	-0.60	-0.81	-1.08	-4.92	-3.28	0.00
3	-10	12	3	2	3	0.015	1.13	1.32	1.48	7.89	3.95	0.00	-1.08	-1.31	-1.53	-7.83	0.00	-3.92
4	-7	9	4	3	6	0.015	1.48	1.56	1.63	9.36	3.12	0.00	-1.53	-1.68	-1.83	-10.08	0.00	-3.36
5	-7	10	5	4	5	0.015	0.00	0.03	0.06	0.16	-0.03	0.00	0.00	-0.05	-0.11	-0.32	0.00	0.05
6	-10	10	6	5	6	0.015	0.06	0.15	0.25	0.90	0.00	0.45	-0.11	-0.28	-0.49	-1.71	-0.86	0.00
7	-10	5	7	6	7	0.015	1.88	1.99	2.01	11.84	9.87	0.00	-2.32	-2.69	-3.07	-16.16	0.00	-13.46
8	-10	0	8	22	7	0.01	0.00	-0.05	-0.08	-0.29	0.10	-0.19	0.00	-0.16	-0.36	-0.98	-0.66	0.33
9	-6	0	9	7	8	0.015	1.93	1.85	1.68	11.01	9.17	0.00	-3.43	-3.80	-4.18	-22.80	0.00	-19.00
10	-3	0	10	8	9	0.015	1.68	1.51	1.33	9.04	0.00	-6.02	-4.18	-4.45	-4.66	-26.61	17.74	0.00
11	0	0	11	23	22	0.01	0.00	-0.04	-0.08	-0.25	0.00	-0.13	0.00	-0.06	-0.14	-0.36	-0.18	0.00
12	3	0	12	22	9	0.02	-0.08	-0.19	-0.34	-1.18	-0.59	0.00	-0.14	-0.32	-0.50	-1.89	0.00	-0.95
13	6	0	13	9	10	0.015	0.99	0.86	0.73	5.17	0.00	-2.58	-5.15	-5.27	-5.35	-31.58	15.79	0.00
14	10	0	14	24	23	0.01	0.00	-0.04	-0.08	-0.25	0.00	-0.13	0.00	-0.01	-0.05	-0.09	-0.05	0.00
15	10	5	15	23	10	0.02	-0.08	-0.19	-0.34	-1.18	-0.59	0.00	-0.05	-0.14	-0.23	-0.81	0.00	-0.41
16	10	10	16	10	11	0.015	0.39	0.26	0.13	1.55	0.00	-0.78	-5.58	-5.63	-5.65	-33.74	16.87	0.00
17	7	10	17	25	24	0.01	0.00	-0.04	-0.08	-0.25	0.00	-0.13	0.00	0.03	0.05	0.18	0.09	0.00
18	7	9	18	24	11	0.02	-0.08	-0.19	-0.34	-1.18	-0.59	0.00	0.05	0.05	0.05	0.27	0.00	0.14
19	10	12	19	11	12	0.015	-0.21	-0.34	-0.47	-2.06	0.00	1.03	-5.60	-5.58	-5.53	-33.47	16.73	0.00
20	10	15	20	26	25	0.01	0.00	-0.04	-0.08	-0.25	0.00	-0.13	0.00	0.08	0.14	0.45	0.23	0.00
21	6	15	21	25	12	0.02	-0.08	-0.19	-0.34	-1.18	-0.59	0.00	0.14	0.23	0.32	1.35	0.00	0.68
22	-6	3	22	12	13	0.015	-0.81	-0.94	-1.07	-5.67	0.00	2.83	-5.22	-5.13	-5.02	-30.77	15.38	0.00
23	-3	3	23	15	26	0.01	0.00	-0.03	-0.08	-0.20	-0.07	-0.13	0.00	0.20	0.36	1.16	0.78	0.39
24	0	3	24	26	13	0.02	-0.08	-0.19	-0.34	-1.16	-0.58	0.00	0.36	0.54	0.72	3.23	0.00	1.61
25	3	3	25	13	14	0.015	-1.41	-1.59	-1.76	-9.52	0.00	6.35	-4.30	-4.09	-3.82	-24.47	16.31	0.00
26	6	3	26	14	15	0.015	-1.76	-1.93	-2.01	-11.49	9.57	0.00	-3.82	-3.44	-3.07	-20.66	0.00	17.21
0	0	0	27	15	16	0.015	-2.01	-1.99	-1.88	-11.84	9.87	0.00	-3.07	-2.69	-2.32	-16.16	0.00	13.46
0	0	0	28	18	17	0.015	0.00	0.03	0.06	0.16	-0.03	0.00	0.00	0.05	0.11	0.32	0.00	-0.05
0	0	0	29	17	16	0.015	0.06	0.15	0.25	0.90	0.00	-0.45	0.11	0.28	0.49	1.71	-0.86	0.00
0	0	0	30	16	19	0.015	-1.63	-1.56	-1.48	-9.36	3.12	0.00	-1.83	-1.68	-1.53	-10.08	0.00	3.36
0	0	0	31	19	20	0.015	-1.48	-1.32	-1.13	-7.89	3.95	0.00	-1.53	-1.31	-1.08	-7.83	0.00	3.92
0	0	0	32	20	21	0.015	-1.13	-0.85	-0.58	-5.13	0.00	-3.42	-1.08	-0.81	-0.60	-4.92	-3.28	0.00
0	0	0	33	21	19	0.015	-0.58	-0.26	0.00	-1.62	-0.81	1.08	-0.60	-0.34	0.00	-1.95	1.30	-0.97
											48.026	**0**					**93.364**	**-3E-14**

Step (3)
After static moment and moment of inertia was calculate, then the calculation of shear flow from bending load was carried out using equation (9). Horizontal shear flow from each nodal coordinates (i, k, j) can be seen in Table 3.

Because the model has close section part, then it has to be corrected by equation (14) to normalized the close form that had been cut, each value from nodal coordinates of shear flow (q) will be integrated by equation (15) to make q^{in}_{cell} (^{in} means initial or before corrected) matrices form or in previous chapter was denoted by $\{\oint(q/t)dS\}$. After matrices q^{in}_{cell} was made especially for multicell type (cell 2 until cell 7), then it has to multiply by the inverse of flexural matrices $[K]^{-1}$ to form $\{q_{corr.}\}=\{q^{in}_{cell}\}\cdot[K]^{-1}$. Correction for single close cell (cell 1 and cell 8) is also made by directly dividing q^{in}_{cell} by $\{\oint dS/t\}$, and all this process can be seen in Table 4.

Important notes for multicell correction:

When the calculation in the matrices form was carried out, then it must be returned the value in each segment as individual correction value as follow:

$$q_{coor\_segment} = q_{coor\_cell}$$

But if one segment used into two cell or more, then it will need recalculation, and it is about the reduction of two q_{coor} that has different direction as follow:

$$q_{coor\_cell} = q_{coor\_cell,\, n} - q_{coor\_cell,\, n+1}$$

Tabel 4 : Shear Flow Correction Procedure

Cell	Seg.	Node i	Node j	∫ ds/t	Σ K	∫ q_y^{in}·ds/t	$q_{by}{}^{in}{}_{cell}$	$q_{by.corr}$	$q_{by.corr}$-i
1	3	2	3	200.00	800.0	2.796	6.298	0.008	0.008
	2	1	2	266.67		2.342			0.008
	1	3	1	333.33		1.160			0.008
2	8	22	7	447.21	1197.2	0.785	27.528	0.026	0.026
	9	7	8	333.33		13.568			0.026
	10	8	9	266.67		12.668			0.026
	12	22	9	150.00		0.506			-0.001
3	11	23	22	300.00	800.0	0.193	12.189	0.025	0.025
	12	22	9	150.00		0.506			-0.001
	13	9	10	200.00		11.274			0.025
	15	23	10	150.00		0.217			-0.001
4	14	24	23	300.00	800.0	0.048	12.238	0.024	0.024
	15	23	10	150.00		0.217			-0.001
	16	10	11	200.00		12.045			0.024
	18	24	11	150.00		-0.072			-0.002
5	17	25	24	300.00	800.0	-0.096	11.418	0.023	0.023
	18	24	11	150.00		-0.072			-0.002
	19	11	12	200.00		11.948			0.023
	21	25	12	150.00		-0.361			-0.002
6	20	26	25	300.00	800.0	-0.241	9.518	0.020	0.020
	21	25	12	150.00		-0.361			-0.002
	22	12	13	200.00		10.985			0.020
	24	26	13	150.00		-0.864			0.001
7	23	15	26	447.21	1197.2	-0.928	22.145	0.021	0.021
	24	26	13	150.00		-0.864			0.001
	25	13	14	266.67		11.646			0.021
	26	14	15	333.33		12.291			0.021
8	31	19	20	200.00	800.0	2.796	6.298	0.008	0.008
	32	20	21	266.67		2.342			0.008
	33	21	19	333.33		1.160			0.008

This individual correction of segment can be seen in Table 4 in the form q_{corr}-i.

Table 3 : Shear Flow of Horizontal Shear Force

Seg.	Node i	Node j	Lk/tk	$q_{by}{}^{in}$-i	$q_{by}{}^{in}$-j	$q_{by}{}^{in}$-k	∫ $q_{by}{}^{n}$·ds/t
1	3	1	333.33	0.0000	0.0036	0.0064	1.160
2	1	2	266.67	0.0064	0.0087	0.0116	2.342
3	2	3	200.00	0.0116	0.0140	0.0164	2.796
4	3	6	133.33	0.0164	0.0180	0.0196	2.399
5	4	5	66.67	0.0000	0.0006	0.0011	0.037
6	5	6	200.00	0.0011	0.0030	0.0052	0.611
7	6	7	333.33	0.0248	0.0288	0.0329	9.613
8	22	7	447.21	0.0000	0.0017	0.0038	0.785
9	7	8	333.33	0.0367	0.0407	0.0447	13.568
10	8	9	266.67	0.0447	0.0476	0.0499	12.668
11	23	22	300.00	0.0000	0.0006	0.0014	0.193
12	22	9	150.00	0.0014	0.0034	0.0053	0.506
13	9	10	200.00	0.0552	0.0564	0.0573	11.274
14	24	23	300.00	0.0000	0.0001	0.0005	0.048
15	23	10	150.00	0.0005	0.0014	0.0024	0.217
16	10	11	200.00	0.0597	0.0603	0.0605	12.045
17	25	24	300.00	0.0000	-0.0004	-0.0005	-0.096
18	24	11	150.00	-0.0005	-0.0005	-0.0005	-0.072
19	11	12	200.00	0.0600	0.0598	0.0593	11.948
20	26	25	300.00	0.0000	-0.0008	-0.0014	-0.241
21	25	12	150.00	-0.0014	-0.0024	-0.0034	-0.361
22	12	13	200.00	0.0559	0.0550	0.0537	10.985
23	15	26	447.21	0.0000	-0.0022	-0.0038	-0.928
24	26	13	150.00	-0.0038	-0.0058	-0.0077	-0.864
25	13	14	266.67	0.0460	0.0438	0.0409	11.646
26	14	15	333.33	0.0409	0.0369	0.0329	12.291
27	15	16	333.33	0.0329	0.0288	0.0248	9.613
28	18	17	66.67	0.0000	-0.0006	-0.0011	-0.037
29	17	16	200.00	-0.0011	-0.0030	-0.0052	-0.611
30	16	19	133.33	0.0196	0.0180	0.0164	2.399
31	19	20	200.00	0.0164	0.0140	0.0116	2.796
32	20	21	266.67	0.0116	0.0087	0.0064	2.342
33	21	19	333.33	0.0064	0.0036	0.0000	1.160

Matrix [K]

	2	3	4	5	6	7
2	1197.21	-150.00	0	0	0	0
3	-150.00	800.00	-150	0	0	0
4	0	-150	800	-150	0	0
5	0	0	-150	800	-150	0
6	0	0	0	-150	800	-150
7	0	0	0	0	-150	1197

Matrik $[K]^{-1}$ × **$\{q_{by}{}^{in}{}_{cell}\}$**

							$\{q_{by}{}^{in}{}_{cell}\}$
8.56E-04	1.67E-04	3.24E-05	6.31E-06	1.21E-06	1.52E-07		27.5276
1.67E-04	1.33E-03	2.59E-04	5.03E-05	9.66E-06	1.21E-06		12.1895
3.24E-05	2.59E-04	1.35E-03	2.62E-04	5.03E-05	6.31E-06	×	12.2377
6.31E-06	5.03E-05	2.62E-04	1.35E-03	2.59E-04	3.24E-05		11.4183
1.21E-06	9.66E-06	5.03E-05	2.59E-04	1.33E-03	1.67E-04		9.5180
1.52E-07	1.21E-06	6.31E-06	3.24E-05	1.67E-04	8.56E-04		22.1446

=

$\{q_{by}{}^{in}{}_{cell}\}\times[K]^{-1}$

$\{q_{by}{}^{in}{}_{cell}\}\times[K]^{-1}$
0.0261
0.0247
0.0242
0.0226
0.0201
0.0210

(Step 4)
When shear flow from bending load was corrected (if the cross section has only pure open part, then it doesn't need correction procedure that has been described in Table 4), then it will continue to the calculation of shear stress according to equation (10). Shear force load taken from Figure 3 and it is about 4740, 51 kN for horizontal shear force, and the result of shear stress value (τ) can be seen in Table 5.

With the same procedure and that about initial shear flow, correction procedure, and calculation of shear stress caused by vertical shear force load has been described in Table 6

Tabel 5 : Shear Stress of Horizontal Direction (τ_{FH})

Seg.	Node i	Node j	Lk/tk	$q_{by}{}^{in}$-i	$q_{by}{}^{in}$-j	$q_{by}{}^{in}$-k	$q_{by.corr}$	q_{by}-i	q_{by}-k	q_{by}-j	τy-i	τy-k	τy-j
1	3	1	333.33	0.0000	0.0036	0.0064	0.0079	-0.0079	-0.0043	-0.0014	-2.49	-1.35	-0.46
2	1	2	266.67	0.0064	0.0087	0.0116	0.0079	-0.0014	0.0008	0.0037	-0.46	0.25	1.17
3	2	3	200.00	0.0116	0.0140	0.0164	0.0079	0.0037	0.0061	0.0085	1.17	1.93	2.69
4	3	6	133.33	0.0164	0.0180	0.0196	0.0000	0.0164	0.0180	0.0196	5.18	5.69	6.19
5	4	5	66.67	0.0000	0.0006	0.0011	0.0000	0.0000	0.0006	0.0011	0.00	0.18	0.36
6	5	6	200.00	0.0011	0.0030	0.0052	0.0000	0.0011	0.0030	0.0052	0.36	0.95	1.65
7	6	7	333.33	0.0248	0.0288	0.0329	0.0000	0.0248	0.0288	0.0329	7.84	9.11	10.38
8	22	7	447.21	0.0000	0.0017	0.0038	0.0261	-0.0261	-0.0244	-0.0223	-12.36	-11.57	-10.55
9	7	8	333.33	0.0367	0.0407	0.0447	0.0261	0.0106	0.0146	0.0186	3.35	4.62	5.89
10	8	9	266.67	0.0447	0.0476	0.0499	0.0261	0.0186	0.0215	0.0238	5.89	6.80	7.52
11	23	22	300.00	0.0000	0.0006	0.0014	0.0247	-0.0247	-0.0241	-0.0232	-11.69	-11.40	-11.00
12	22	9	150.00	0.0014	0.0034	0.0053	-0.0014	0.0029	0.0048	0.0067	0.68	1.14	1.59
13	9	10	200.00	0.0552	0.0564	0.0573	0.0247	0.0305	0.0318	0.0327	9.64	10.04	10.33
14	24	23	300.00	0.0000	0.0001	0.0005	0.0242	-0.0242	-0.0240	-0.0237	-11.45	-11.39	-11.22
15	23	10	150.00	0.0005	0.0014	0.0024	-0.0005	0.0010	0.0020	0.0029	0.23	0.46	0.69
16	10	11	200.00	0.0597	0.0603	0.0605	0.0242	0.0356	0.0361	0.0363	11.25	11.42	11.48
17	25	24	300.00	0.0000	-0.0004	-0.0005	0.0226	-0.0226	-0.0229	-0.0230	-10.70	-10.87	-10.92
18	24	11	150.00	-0.0005	-0.0005	-0.0005	-0.0016	0.0011	0.0011	0.0011	0.26	0.26	0.26
19	11	12	200.00	0.0600	0.0598	0.0593	0.0226	0.0374	0.0372	0.0367	11.83	11.77	11.60
20	26	25	300.00	0.0000	-0.0008	-0.0014	0.0201	-0.0201	-0.0209	-0.0215	-9.51	-9.91	-10.20
21	25	12	150.00	-0.0014	-0.0024	-0.0034	-0.0025	0.0011	0.0001	-0.0009	0.25	0.02	-0.21
22	12	13	200.00	0.0559	0.0550	0.0537	0.0201	0.0358	0.0349	0.0336	11.32	11.03	10.63
23	15	26	447.21	0.0000	-0.0022	-0.0038	0.0210	-0.0210	-0.0232	-0.0248	-9.96	-10.98	-11.78
24	26	13	150.00	-0.0038	-0.0058	-0.0077	0.0009	-0.0048	-0.0067	-0.0086	-1.13	-1.59	-2.05
25	13	14	266.67	0.0460	0.0438	0.0409	0.0210	0.0250	0.0228	0.0199	7.91	7.20	6.28
26	14	15	333.33	0.0409	0.0369	0.0329	0.0210	0.0199	0.0159	0.0118	6.28	5.01	3.74
27	15	16	333.33	0.0329	0.0288	0.0248	0.0000	0.0329	0.0288	0.0248	10.38	9.11	7.84
28	18	17	66.67	0.0000	-0.0006	-0.0011	0.0000	0.0000	-0.0006	-0.0011	0.00	-0.18	-0.36
29	17	16	200.00	-0.0011	-0.0030	-0.0052	0.0000	-0.0011	-0.0030	-0.0052	-0.36	-0.95	-1.65
30	16	19	133.33	0.0196	0.0180	0.0164	0.0000	0.0196	0.0180	0.0164	6.19	5.69	5.18
31	19	20	200.00	0.0164	0.0140	0.0116	0.0079	0.0085	0.0061	0.0037	2.69	1.93	1.17
32	20	21	266.67	0.0116	0.0087	0.0064	0.0079	0.0037	0.0008	-0.0014	1.17	0.25	-0.46
33	21	19	333.33	0.0064	0.0036	0.0000	0.0079	-0.0014	-0.0043	-0.0079	-0.46	-1.35	-2.49

Step (5)
The calculation of shear stress caused by bending load has been finished, and it will be follow by numerical procedure for shear stress calculation caused by torsional load. The calculation procedure begin with defined normalized warping function ὼn that called from Part-1 study, with equation (11) warping static torsion Sὼ flow in accordance with figure 2 and follow symbol of "flow ⇀".

Warping static flow Sὼ will be zero (0) in the free end either free end from open part or free end from cutting (×) procedure for close part. When two or more of flow meets, then it will be added as water flow process. This flow starting from the meeting point of centreline (CL) and top part of model, and follow the outside part of model until it reach again to where it come from, which can be seen in figure 2 symbol "path of flow ⇢"

The calculation of warping constant can be perform in two method, at firs by normalized warping function as can be seen in Part-1 study table 5, and the second by using warping static moment Sὼ as the component and the formula for calculation will follow :

$$I\grave{\omega} = \int S\grave{\omega}.d\grave{\omega} \qquad (16)$$

For calculate Iὼ, it have to begin with calculate product of Sὼ in related to do numerical procedure, product of Sὼ follow Simpson method and that will be :

$$S\grave{\omega}\text{-}i + 4\cdot S\grave{\omega}\text{-}k + S\grave{\omega}\text{-}j.$$

To completed the procedure, then it will be multiplied with 1/6 ὼc, ὼc was DSA with respect to shear centre after corrected by close part and it was described briefly in step (4) until step (5) Part-1 study. The final value of Iὼ is same although the method for used is different, and the value will be 4412,41 m^6 as can be seen in Table 7.

Table 6 : Shear Stress of Vertical Direction (τ_{FV})

Seg.	Node i	Node j	Lk/tk	q_{bx}^{in}-i	q_{bx}^{in}-j	q_{bx}^{in}-k	$\int q_{bx}^{n} \cdot ds/t$	$q_{bx.corr}$	q_{bx}-i	q_{bx}-k	q_{bx}-j	τx-i	τx-k	τx-j
1	3	1	333.33	0.0000	-0.0054	-0.0120	-1.876	-0.0151	-0.0151	-0.0097	-0.0031	-5.83	-3.73	-1.19
2	1	2	266.67	-0.0120	-0.0178	-0.0235	-4.744	-0.0151	-0.0031	0.0027	0.0084	-1.19	1.03	3.25
3	2	3	200.00	-0.0235	-0.0275	-0.0308	-5.478	-0.0151	0.0084	0.0124	0.0156	3.25	4.77	6.03
4	3	6	133.33	-0.0308	-0.0325	-0.0340	-4.333	0.0000	0.0308	0.0325	0.0340	11.86	12.54	13.11
5	4	5	66.67	0.0000	-0.0005	-0.0012	-0.037	0.0000	0.0000	0.0005	0.0012	0.00	0.21	0.45
6	5	6	200.00	-0.0012	-0.0031	-0.0051	-0.626	0.0000	0.0012	0.0031	0.0051	0.45	1.21	1.97
7	6	7	333.33	-0.0391	-0.0414	-0.0418	-13.702	0.0000	0.0391	0.0414	0.0418	15.08	15.97	16.11
8	22	7	447.21	0.0000	0.0011	0.0017	0.442	-0.0175	-0.0175	-0.0185	-0.0191	-10.09	-10.71	-11.06
9	7	8	333.33	-0.0401	-0.0385	-0.0350	-12.734	-0.0175	0.0227	0.0211	0.0175	8.73	8.12	6.75
10	8	9	266.67	-0.0350	-0.0314	-0.0277	-8.363	-0.0175	0.0175	0.0139	0.0103	6.75	5.36	3.96
11	23	22	300.00	0.0000	0.0009	0.0017	0.262	-0.0058	-0.0058	-0.0067	-0.0075	-3.34	-3.85	-4.35
12	22	9	150.00	0.0017	0.0040	0.0071	0.617	0.0117	0.0099	0.0077	0.0046	2.87	2.23	1.32
13	9	10	200.00	-0.0206	-0.0179	-0.0152	-3.585	-0.0058	0.0149	0.0121	0.0094	5.73	4.68	3.63
14	24	23	300.00	0.0000	0.0009	0.0017	0.262	0.0006	0.0006	-0.0003	-0.0012	0.32	-0.18	-0.69
15	23	10	150.00	0.0017	0.0040	0.0071	0.617	0.0063	0.0046	0.0024	-0.0008	1.33	0.69	-0.22
16	10	11	200.00	-0.0081	-0.0054	-0.0027	-1.079	0.0006	0.0087	0.0059	0.0032	3.34	2.29	1.25
17	25	24	300.00	0.0000	0.0009	0.0017	0.262	0.0060	0.0060	0.0051	0.0042	3.45	2.94	2.44
18	24	11	150.00	0.0017	0.0040	0.0071	0.617	0.0054	0.0037	0.0015	-0.0017	1.06	0.42	-0.49
19	11	12	200.00	0.0044	0.0071	0.0099	1.428	0.0060	0.0015	-0.0012	-0.0039	0.59	-0.45	-1.50
20	26	25	300.00	0.0000	0.0009	0.0017	0.262	0.0118	0.0118	0.0109	0.0100	6.80	6.30	5.80
21	25	12	150.00	0.0017	0.0040	0.0071	0.617	0.0058	0.0041	0.0018	-0.0013	1.17	0.53	-0.38
22	12	13	200.00	0.0170	0.0197	0.0224	3.934	0.0118	-0.0052	-0.0079	-0.0106	-2.00	-3.04	-4.09
23	15	26	447.21	0.0000	0.0006	0.0017	0.304	0.0207	0.0207	0.0201	0.0190	11.96	11.61	11.00
24	26	13	150.00	0.0017	0.0039	0.0070	0.605	0.0089	0.0073	0.0050	0.0019	2.10	1.46	0.55
25	13	14	266.67	0.0294	0.0330	0.0366	8.808	0.0207	-0.0087	-0.0123	-0.0160	-3.36	-4.76	-6.15
26	14	15	333.33	0.0366	0.0402	0.0418	13.290	0.0207	-0.0160	-0.0195	-0.0211	-6.15	-7.52	-8.13
27	15	16	333.33	0.0418	0.0414	0.0391	13.702	0.0000	-0.0418	-0.0414	-0.0391	-16.11	-15.97	-15.08
28	18	17	66.67	0.0000	-0.0005	-0.0012	-0.037	0.0000	0.0000	0.0005	0.0012	0.00	0.21	0.45
29	17	16	200.00	-0.0012	-0.0031	-0.0051	-0.626	0.0000	0.0012	0.0031	0.0051	0.45	1.21	1.97
30	16	19	133.33	0.0340	0.0325	0.0308	4.333	0.0000	-0.0340	-0.0325	-0.0308	-13.11	-12.54	-11.86
31	19	20	200.00	0.0308	0.0275	0.0235	5.478	0.0151	-0.0156	-0.0124	-0.0084	-6.03	-4.77	-3.25
32	20	21	266.67	0.0235	0.0178	0.0120	4.744	0.0151	-0.0084	-0.0027	0.0031	-3.25	-1.03	1.19
33	21	19	333.33	0.0120	0.0054	0.0000	1.876	0.0151	0.0031	0.0097	0.0151	1.19	3.73	5.83

Tabel 7 : Warping Static Moment (Sώ) and Warping Constant (Iώ)

Seg.	Node i	Node j	ώni	ώnk	ώnj	Sώi [m^4]	Sώk [m^4]	Sώj [m^4]	Product-Sώ	Iώ [m^6]
1	3	1	-29.00	-82.13	-135.26	0.00	-2.08	-6.16	-14.49	256.71
2	1	2	-135.26	-95.63	-56.00	-6.16	-9.62	-11.90	-56.55	-747.09
3	2	3	-56.00	-42.50	-29.00	-11.90	-13.01	-13.81	-77.73	-349.78
4	3	6	-29.00	-19.00	-9.00	-13.81	-14.17	-14.38	-84.87	-282.89
5	4	5	-49.45	-52.95	-56.45	0.00	-0.38	-0.79	-2.33	2.72
6	5	6	-56.45	-32.72	-9.00	-0.79	-1.80	-2.27	-10.25	-81.06
7	6	7	-9.00	16.00	41.00	-16.65	-16.51	-15.45	-98.15	-817.93
8	22	7	31.65	36.33	41.00	0.00	0.76	1.62	4.66	7.27
9	7	8	41.00	60.82	80.63	-13.82	-11.91	-9.26	-70.73	-467.13
10	8	9	80.63	64.85	49.07	-9.26	-7.08	-5.37	-42.94	225.91
11	23	22	15.72	23.69	31.65	0.00	0.30	0.71	1.89	5.03
12	22	9	31.65	40.36	49.07	0.71	1.79	3.13	11.01	31.94
13	9	10	49.07	36.84	24.61	-2.24	-1.27	-0.58	-7.90	32.20
14	24	23	0.00	7.86	15.72	0.00	0.06	0.24	0.47	1.24
15	23	10	15.72	20.16	24.61	0.24	0.77	1.45	4.78	7.08
16	10	11	24.61	12.30	0.00	0.87	1.28	1.42	7.41	-30.40
17	25	24	-15.72	-7.86	0.00	0.00	-0.18	-0.24	-0.94	-2.47
18	24	11	0.00	0.00	0.00	-0.24	-0.24	-0.24	-1.41	0.00
19	11	12	0.00	-12.30	-24.61	1.18	1.05	0.63	6.00	-24.59
20	26	25	-31.65	-23.69	-15.72	0.00	-0.42	-0.71	-2.37	-6.30
21	25	12	-15.72	-20.16	-24.61	-0.71	-1.25	-1.92	-7.63	11.30
22	12	13	-24.61	-36.84	-49.07	-1.29	-1.98	-2.95	-12.16	49.58
23	15	26	-41.00	-36.33	-31.65	0.00	-0.86	-1.62	-5.08	-7.92
24	26	13	-31.65	-40.36	-49.07	-1.62	-2.70	-4.05	-16.49	47.85
25	13	14	-49.07	-64.85	-80.63	-6.99	-8.70	-10.88	-52.69	277.19
26	14	15	-80.63	-60.82	-41.00	-10.88	-13.54	-15.45	-80.48	-531.51
27	15	16	-41.00	-16.00	9.00	-15.45	-16.51	-16.65	-98.15	-817.93
28	18	17	49.45	52.95	56.45	0.00	0.38	0.79	2.33	2.72
29	17	16	56.45	32.72	9.00	0.79	1.80	2.27	10.25	-81.06
30	16	19	9.00	19.00	29.00	-14.38	-14.17	-13.81	-84.87	-282.89
31	19	20	29.00	42.50	56.00	-13.81	-13.01	-11.90	-77.73	-349.78
32	20	21	56.00	95.63	135.26	-11.90	-9.62	-6.16	-56.55	-747.09
33	21	19	135.26	82.13	29.00	-6.16	-2.08	0.00	-14.49	256.71
				135.26						**4412.41**

Step (6)
After finished describe $S_{\dot{\omega}}$ and $I_{\dot{\omega}}$, then it will follow by calculate shear flow ($q_{\dot{\omega}}$) from torsional load according to equation (12) for each nodal coordinates. If the model has close part then it have to be corrected like the procedure described in step (3). After shear flow had been corrected then it will multiplied by $T_{\dot{\omega}}/t$, where $T_{\dot{\omega}}$ is the torsional moment which can be seen in Figure 3 and for midship area the value will be 63386,43 kNm, and t is the thickness of segment. The final result of shear stress caused by wave torsional load ($\tau_{\dot{\omega}}$) calculation presented in Table 8.

Table 8 : Warping shear Stress

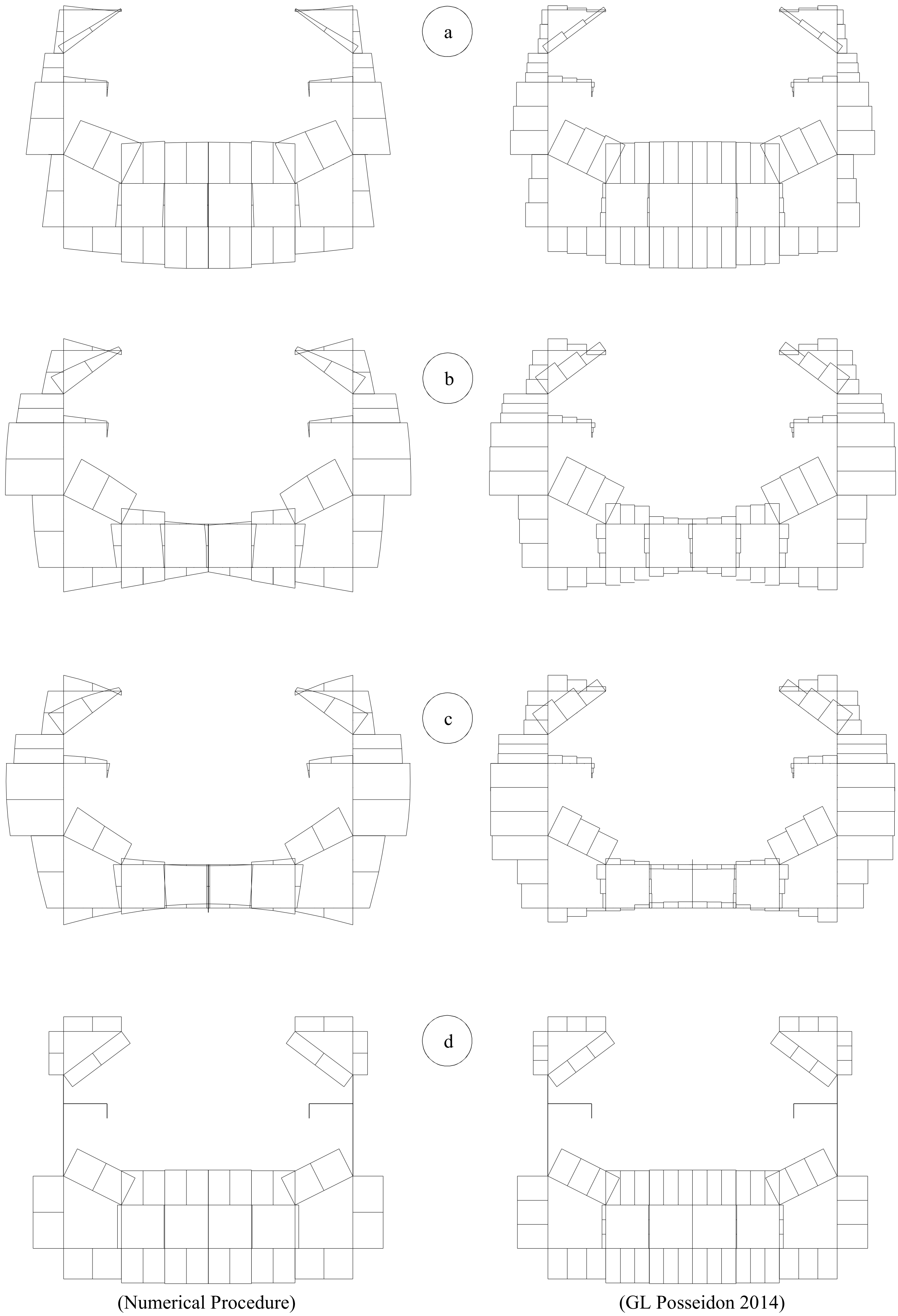
Seg.	Node i	Node j	Lk/tk	$q_{\dot{\omega}}^{in}$-i	$q_{\dot{\omega}}^{in}$-k	$q_{\dot{\omega}}^{in}$-j	$\int q_{\dot{\omega}}^{in}\cdot ds/t$	$q_{\dot{\omega}.corr}$	$q_{\dot{\omega}}$-i	$q_{\dot{\omega}}$-k	$q_{\dot{\omega}}$-j	$\tau_{\dot{\omega}}$-i	$\tau_{\dot{\omega}}$-k	$\tau_{\dot{\omega}}$-j
1	3	1	333.33	0.0000	0.0005	0.0014	0.182	0.0017	-0.0017	-0.0012	-0.0003	-14.15	-10.16	-2.35
2	1	2	266.67	0.0014	0.0022	0.0027	0.570	0.0017	-0.0003	0.0005	0.0010	-2.35	4.28	8.64
3	2	3	200.00	0.0027	0.0029	0.0031	0.587	0.0017	0.0010	0.0013	0.0015	8.64	10.76	12.30
4	3	6	133.33	0.0031	0.0032	0.0033	0.427	0.0000	0.0031	0.0032	0.0033	26.45	27.14	27.54
5	4	5	66.67	0.0000	0.0001	0.0002	0.006	0.0000	0.0000	0.0001	0.0002	0.00	0.74	1.52
6	5	6	200.00	0.0002	0.0004	0.0005	0.077	0.0000	0.0002	0.0004	0.0005	1.52	3.44	4.34
7	6	7	333.33	0.0038	0.0037	0.0035	1.236	0.0000	0.0038	0.0037	0.0035	31.88	31.63	29.59
8	22	7	447.21	0.0000	-0.0002	-0.0004	-0.079	0.0010	-0.0010	-0.0012	-0.0014	-12.70	-14.88	-17.36
9	7	8	333.33	0.0031	0.0027	0.0021	0.891	0.0010	0.0021	0.0017	0.0011	18.01	14.35	9.27
10	8	9	266.67	0.0021	0.0016	0.0012	0.433	0.0010	0.0011	0.0006	0.0002	9.27	5.09	1.82
11	23	22	300.00	0.0000	-0.0001	-0.0002	-0.021	0.0001	-0.0001	-0.0002	-0.0003	-1.44	-2.29	-3.48
12	22	9	150.00	-0.0002	-0.0004	-0.0007	-0.062	-0.0009	0.0007	0.0005	0.0002	4.61	3.06	1.13
13	9	10	200.00	0.0005	0.0003	0.0001	0.060	0.0001	0.0004	0.0002	0.0000	3.33	1.47	0.15
14	24	23	300.00	0.0000	0.0000	-0.0001	-0.005	-0.0001	0.0001	0.0000	0.0000	0.70	0.53	0.02
15	23	10	150.00	-0.0001	-0.0002	-0.0003	-0.027	-0.0002	0.0001	0.0000	-0.0002	0.73	-0.04	-1.01
16	10	11	200.00	-0.0002	-0.0003	-0.0003	-0.056	-0.0001	-0.0001	-0.0002	-0.0003	-1.19	-1.99	-2.25
17	25	24	300.00	0.0000	0.0000	0.0001	0.011	0.0001	-0.0001	-0.0001	-0.0001	-1.62	-1.12	-0.95
18	24	11	150.00	0.0001	0.0001	0.0001	0.008	0.0002	-0.0001	-0.0001	-0.0001	-0.82	-0.82	-0.82
19	11	12	200.00	-0.0003	-0.0002	-0.0001	-0.045	0.0001	-0.0004	-0.0004	-0.0003	-3.35	-3.09	-2.29
20	26	25	300.00	0.0000	0.0001	0.0002	0.027	0.0006	-0.0006	-0.0005	-0.0005	-7.96	-6.77	-5.92
21	25	12	150.00	0.0002	0.0003	0.0004	0.043	0.0005	-0.0003	-0.0002	-0.0001	-2.15	-1.37	-0.41
22	12	13	200.00	0.0003	0.0004	0.0007	0.092	0.0006	-0.0003	-0.0002	0.0000	-2.84	-1.51	0.34
23	15	26	447.21	0.0000	0.0002	0.0004	0.086	0.0015	-0.0015	-0.0013	-0.0011	-19.25	-16.76	-14.58
24	26	13	150.00	0.0004	0.0006	0.0009	0.093	0.0009	-0.0005	-0.0003	0.0000	-3.31	-1.76	0.17
25	13	14	266.67	0.0016	0.0020	0.0025	0.531	0.0015	0.0001	0.0005	0.0009	0.57	3.84	8.02
26	14	15	333.33	0.0025	0.0031	0.0035	1.013	0.0015	0.0009	0.0015	0.0020	8.02	13.10	16.76
27	15	16	333.33	0.0035	0.0037	0.0038	1.236	0.0000	0.0035	0.0037	0.0038	29.59	31.63	31.88
28	18	17	66.67	0.0000	-0.0001	-0.0002	-0.006	0.0000	0.0000	-0.0001	-0.0002	0.00	-0.74	-1.52
29	17	16	200.00	-0.0002	-0.0004	-0.0005	-0.077	0.0000	-0.0002	-0.0004	-0.0005	-1.52	-3.44	-4.34
30	16	19	133.33	0.0033	0.0032	0.0031	0.427	0.0000	0.0033	0.0032	0.0031	27.54	27.14	26.45
31	19	20	200.00	0.0031	0.0029	0.0027	0.587	0.0017	0.0015	0.0013	0.0010	12.30	10.76	8.64
32	20	21	266.67	0.0027	0.0022	0.0014	0.570	0.0017	0.0010	0.0005	-0.0003	8.64	4.28	-2.35
33	21	19	333.33	0.0014	0.0005	0.0000	0.182	0.0017	-0.0003	-0.0012	-0.0017	-2.35	-10.16	-14.15

5. VALIDATION

To verify the numerical procedure that have been made, the calculation result will be compared with commercial software GL Posseidon 2014. Shear stress distribution caused by either vertical shear force, or horizontal shear force or torsional load from purpose numerical procedure will be compare virtually, and that by comparing graphical result for each shear stresses. Figure 4 (a) represented shear stress by horizontal shear force, Figure 4 (b) represented shear stress by vertical shear force, Figure 4 (c) represented warping shear stress by torsional load, and Figure 4 (d) represented st.venant shear stress by torsional load.

Comparison of shear stress caused by horizontal shear force in Figure 4 (a) shows almost no different between purpose numerical procedure and Posseidon, in part located near centreline have greatest value of shear stress and that happen because the normal stress was minimum in those location. Figure 4 (b) shows a similar result between two procedure, and the greatest value of shear stress happened around of neutral axis (NA) of the cross section. This is suitable with the fact that maximum shear stress occurred when normal stress takes the lowest values, and for case both of sagging and hogging takes maximum normal stress either in deck or bottom, end will be zero in neutral axis location. Shear stress calculation needed at least three values on one segment, because the distribution form always makes curvature and have some radius therefore it is very different with normal stress that always in linier form.

Comparison of warping shear stress caused by torsional load shows a bit different as can be seen in Figure 4 (c), and that happen in about double bottom area near centreline, but this bit different will takes no effect since the greatest value of shear stress occurred around neutral axis, so this different will be neglected. And the last is comparison in Figure 4 (d), st.venant shear stress effect, and it is shows perfect similarity. Therefore successfully comparison was reached as illustrated in Figure 4 (a)-(d).



(Numerical Procedure) (GL Posseidon 2014)

Figure 4 (a),(b),(c),(d)

6. CONCLUSION

Practical solution for evaluating shear stress distribution with detailed step by step numerical procedure was described briefly in this study and applied to complex combination of ship cross section including its verification with GL Poseidon 2014. There are several important point can be concluded:

- This study provides a detailed procedure for computing the shear stress distribution of ship's cross section with combination of open, single closed section and multi cell. This procedure is provided to help practical engineers calculate (τ_{FH}, τ_{FV}, τ_{ω}, $\tau_{ST.VENANT}$) values for ship cross section including hull girder load from BKI Rule for Hull II, Section 5, 2014.
- Combination cases between normal stress distribution that had been described in Part-1 study and shear stress distribution in this study are also needed for determination the most unfavourable combination stresses values in order to avoid local structural failure due to hull girder loads.
- The purpose procedure could be applied to any platform of software calculation, e.g. Ms Excel, Mathcad, Matlab, etc.
- In order to improve accuracy of the result, the previous numerical procedure calculation can be applied with combination both of plate and stiffener.

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